

Information Theory: Exercise III

- 1) Prove that the two definitions that we gave for the capacity of the symmetric binary channel coincide. That is, the capacity of the channel according to the general definition is $1 - H(p, 1 - p)$.
- 2) Consider the following channel: on input $x \in \{0, 1\}$ the channel outputs x with probability $1 - p$ and a special symbol (say, $*$) with probability p . What is the capacity of the channel ?
- 3) Consider the following channel: on input $(x_1, \dots, x_n) \in \{0, 1\}^n$ the channel erases exactly one of the bits x_1, \dots, x_n (with probability $1/n$ each) and outputs the sequence of all the other bits. Show that for any n the capacity of the channel is at least $n - \log n - 1$. What is the capacity of the channel for $n = 2$?
- 4) Let Q_1, Q_2 be two arbitrary channels. Let Q be the combined channel, that is, on input (x_1, x_2) where x_1 is an input for Q_1 and x_2 is an input for Q_2 , the channel Q applies the channel Q_1 on x_1 and Q_2 on x_2 . What is the capacity of the channel Q (as a function of the capacities of Q_1, Q_2) ?